Enhancement of cooperation in highly clustered scale-free networks.

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We study the effect of clustering on the organization of cooperation, by analyzing the evolutionary dynamics of the Prisoner's Dilemma on scale-free networks with a tunable value of clustering. We find that a high value of the clustering coefficient produces an overall enhancement of cooperation in the network, even for a very high temptation to defect. On the other hand, high clustering homogeneizes the process of invasion of degree classes by defectors, decreasing the chances of survival of low densities of cooperator strategists in the network.

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Cooperative phenomena are essential in natural and human systems and have been the subject of intense research during decades [1, 2, 3, 4, 5, 6]. Evolutionary game theory is concerned with systems of replicating agents programmed to use some strategy in their interactions with other agents, which ultimately yields a feedback loop that drives the evolution of the strategies composition of the population [6, 7, 8]. To understand the observed survival of cooperation among unrelated individuals in populations when selfish actions provide a short-term higher benefit, a lot of attention has been paid to the analysis of evolutionary dynamics of the *Prisoner's Dilemma* (PD) game. In this simple two-players game, individuals adopt one of the two available strategies, cooperation (C) or defection (D); both receive R under mutual cooperation and P under mutual defection, while a cooperator receives S when confronted to a defector, which in turn receives T, where $T > R > P > S$. Under these conditions in a one-shot game it is better to defect, regardless of the opponent strategy, and the proportion of cooperators asymptotically vanishes in a well-mixed population. On the other hand, the structure of interactions among individuals in real societies are seen to be described by complex networks of contacts rather than by a set of agents connected all-to-all [9, 10]. Therefore, it is necessary to abandon the panmixia hypothesis to study how cooperative behavior appear in the social context.

Several studies [11, 12, 13, 14, 15, 16, 17, 18, 19] have reported the asymptotic survival of cooperation on different kinds of networks. Notably, cooperation even dominates over defection in non-homogeneous, *scale-free* (SF) networks, i.e. in graphs where the number k of neighbors of an individual (the node degree) is distributed as a power law [12, 15], $P(k) \sim k^{-\gamma}$, with $2 < \gamma \leq 3$. Networks with such a distribution are ubiquitous: scale-free topologies appear as the backbone of many social, biological, technological complex systems. However, in the context of social systems, other topological features, such as the presence of degree-degree correlations and of high clustering coefficients, are relevant ingredients to take into account in a complete description of the networks. The studies of the PD game on SF networks have considered so far networks with no degree correlations and nearly zero clustering coefficient, with the remarkable exception of Ref. [20] where high clustering SF networks are studied. Therefore, it is necessary to explore the effects that structural properties such as clustering and degre-degree correlations have on the survival of cooperation in complex netwoks.

In this paper, we focus on the effects that the presence of non vanishing clustering coefficient have on the survival of cooperation. The clustering coefficient of a network is related to the number of triangles present in the network, and is defined as the probability that two neighbors of a given node share also a connection between them [9, 10]. A high clustering coefficient points out the presence of local neighborhoods, *i.e.* small clusters of densely interconnected nodes, in the network. This property is present in most of social networks where two friends of an individual are also friends with high probability. Therefore a full description of cooperative phenomena in social networks should be tackled by considering highly clustered scale-free networks.

Network model.- We study a class of SF networks with a tunable clustering coefficient introduced by Holme and Kim (HK) in Ref. [21]. The networks are constructed via a growing process that starts from an initial core of m_0 unconnected nodes. At each time step, a new node i ($i = m_0 + 1, ..., N$) is added to the network and links to m (with $m \leq m_0$) of the previously existent nodes. The first link follows a preferential attachment rule (PA), *i.e.* the probability that node i attaches to a node j of the network (with $j < i$) is proportional to the degree k_j of the node j. The remaining $m - 1$ links are attached in two different ways: *(i)* with probability p the new node i is connected to a randomly chosen neighbor of node j; *(ii)* with probability $(1-p)$ the PA rule is used again, and node i is connected to another one of the previously existent nodes. With such a procedure one obtains SF networks with degree distribution $P(k) \sim k^{-3}$, and a tunable clustering coefficient depending on the value of p. In particular, for $p = 0$ we recover the Barabási-Albert model [22] where the clustering coeffienct tends to zero as the network size N goes to infinity. For values of $p > 0$ the clustering coefficient monotonously grows with p [21].

We have first checked that the networks produced by the HK model have no degree-degree correlations, and we have

analyzed the dependence of the node clustering coefficient on the node degree. The clustering coefficient of a node i, CC_i, expresses how likely $a_{jm} = 1$ for two neighbors j and m of node i, where $A = \{a_{ij}\}$ is the adjacency matrix of the graph. The value of CC_i is obtained by counting the actual number of edges, denoted by e_i, in G_i, the subgraph of neighbors of i. The clustering coefficient of i is defined as the ratio between e_i and $k_i(k_i - 1)/2$, the maximum possible number of edges in G_i [9, 10]:

$$CC_i = \frac{2e_i}{k_i(k_i-1)} = \frac{\sum_{j,m} a_{ij}a_{jm}a_{mi}}{k_i(k_i-1)} \,. \tag{1}$$

The mean clustering coefficient of the graph, CC, is then given by the average of CC_i over all the nodes in the network. By definition, $0 \leq CC_i \leq 1$ and $0 \leq CC \leq 1$. In Fig. 1 we report the results obtained for networks with $m = m_0 = 3$ and $N = 5 \cdot 10^3$. We have considered different values of p corresponding to networks with mean clustering coefficient $CC = 0, 0.1, 0.2, 0.33, 0.46$ and 0.65. Ensembles of $2 \cdot 10^4$ networks have been generated for each value of p. In Fig. 1.a we plot, as a function of k, the average degree $K_{nn}(k)$ of the neighbors of nodes with degree k. The figure shows a nearly constant function $K_{nn}(k)$, pointing out that the HK model produces SF networks with no degree-degree correlations. This result is further confirmed by computing the assortative index r, introduced in Ref. [23], as a punction of the networks' CC. As observed from Fig. 1.b the values of r are close to 0 for all values of the CC, thus confirming the absence of degree-degree correlations in all the studied networks. On the other hand, Fig. 1.c reveals that the average clustering coefficient $CC(k)$ of nodes with degree k, strongly depends on k. In particular we observe a power law decay $CC(k) \sim k^{-\alpha}$ for high values of the mean clustering coefficient of the network. Therefore, all the networks used in this work have the same degree distribution and no degree-degree correlations and thus they allow us to make a correct estimate of the role of the clustering coefficient on the promotion of cooperation in SF networks.

Evolutionary Dynamics.- We now assume that each node of the graph represents a player. A link between two nodes of the graph indicates that the two players interact and can play. We implement the finite population analogue of replicator dynamics [12, 15] for the PD game with payoffs $R = 1$, $P = S = 0$, and $T = b > 1$. At each generation, of the discrete evolutionary time, t, each agent i plays once with every agent in its neighborhood and accumulates the obtained payoffs, P_i. Then all the players update synchronously their strategies by the following rules. Each individual i chooses at random a neighbor, j, and compares its payoff P_i with P_j. If $P_i \geq P_j$, player i keeps the same strategy for the next generation. On the other hand, if $P_j > P_i$, the player i adopts the strategy of its neighbor j with probability $\Pi_{i\to j} = \beta(P_j - P_i)$, for the next game round robin. Here, β is related to the characteristic inverse time scale: the larger β, the faster evolution takes place. We assume $\beta = (\max\{k_i, k_j\}b)^{-1}$. This choice

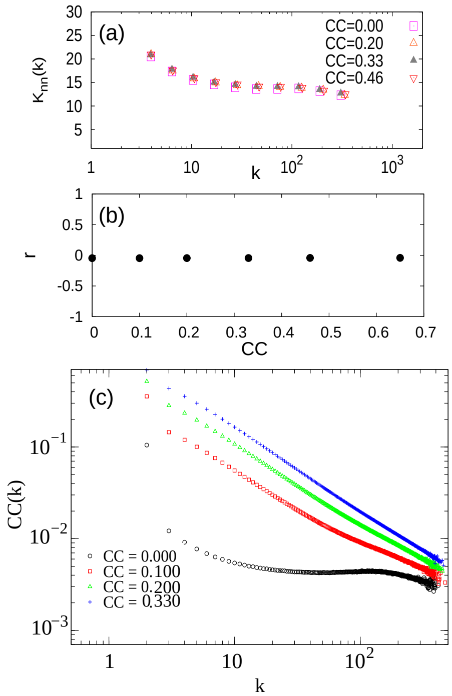

FIG. 1: (Color online). **(a)** Average degree of the neighbors of nodes with degree k, $K_{nn}(k)$, for four SF networks with different values of the CC. **(b)** Assortative index , r, as a function of the CC of the networks. Both measures, $K_{nn}(k)$ and r, reveal that SF networks generated from the HK model show no degree-degree correlations. **(c)** Mean clustering coefficient of nodes with degree k, $CC(k)$, for four SF networks with different CC. From this figure it is clear a power law decay, $CC(k) \sim k^{-\alpha}$, for highly clustered networks.

assures that $\Pi_{i\to j} < 1$ and also slows down the invasion process from or to highly connected nodes [12].

After a transient time, the evolutionary dynamics reaches a stationary regime which can be characterized by the average cooperation index $\langle c\rangle$, defined as the overall fraction of time spent by all the players in the cooperator state. The value of $\langle c\rangle$ is computed as follows. After a transient time $\tau_0 = 5\cdot 10^3$, we further evolve the system over time windows of $\tau = 10^3$ generations each, and we study the time evolution of the number of cooperators, $c(t)$. In each time window we compute the average value and the fluctuations of $c(t)$. When the fluctuations are less or equal to $1/\sqrt{N}$, we stop the simulation and we consider the average cooperation obtained in the last time window, as the asymptotic average cooperation $\langle c\rangle$ of the realization. In each realization we change both the network and the initial conditions of the dynamics. All the results reported below are averages over 10^3 realizations for each value of the network and game parameters (p and b respectively).

Results.- To unveil the influence that clustering has on the promotion of cooperation in scale-free networks, we explore the evolutionary dynamics on networks with different values of the clustering coefficient. In Fig. 2 we report $\langle c\rangle$ as a function of b. As expected, the degree of cooperation $\langle c\rangle$ decreases monotonously as the temptation to defect b increases. How-

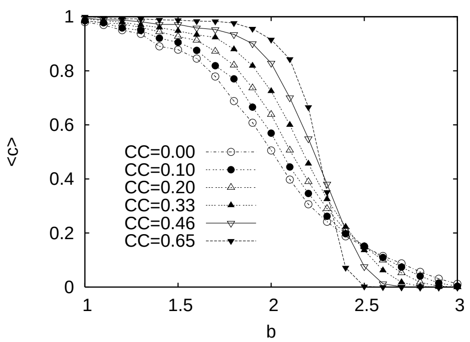

FIG. 2: Average degree of cooperation $\langle c \rangle$ as a function of the temptation to defect b. The six different curves show the transition from all-cooperator to all-defector states for SF networks with different average clustering coefficient. On the one hand, the cooperation is enhanced as the clustering coefficient increases. On the other hand, the transition to all-defector networks is smoother when clustering is smaller.

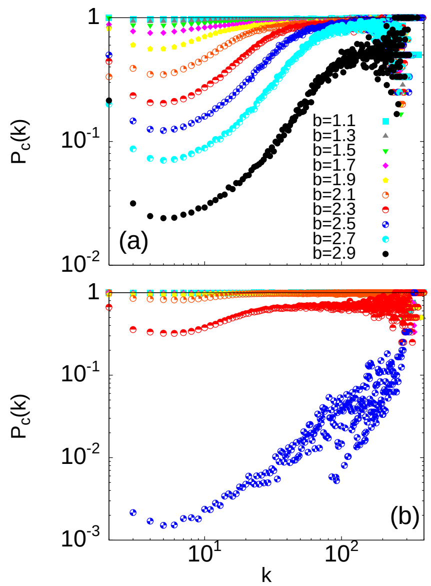

FIG. 3: Probability $P_c(k)$ of finding a node of degree k playing as cooperator in the stationary regime of the evolutionary dynamics. Different curves correspond to different values of the temptation to defect, b. The two panels correpond to two SF with different CC, namely **(a)** $CC = 0.0$, **(b)** $CC = 0.65$.

ever, the path from an all-cooperator network, at $b = 1$, to an all-defector network, for high values of b, depends strongly on the clustering coefficient of the SF network. From the figure it is clear that SF networks with the highest clustering coefficient show a remarkable survival of cooperation with values $\langle c \rangle \simeq 1$ up to temptation values of $b = 2$, in agreement with [20]. This is in contrast with the constant decrease of the cooperation observed in networks with no clustering. On the other hand, the enhancement of cooperation for clustered SF networks disappears when moving to higher values of b. In particular, a sharp decrease from high to zero cooperation is observed when b varies in the narrow range $b \in (2, 2.5)$, with SF networks with small custering coefficients showing a slower convergence to the all-defector state.

Since all the networks analyzed share the same degree distribution, it is possible to compare the microscopic evolution of cooperation as a function of b by looking at the probability, $P_c(k)$, that a node of degree k acts as cooperator in the stationary regime. Such a probability is calculated by considering the final time configurations for each value of b and p. Namely, for a given realization l (of the network and of the initial conditions), we measure the final number $c_l(k)$ of cooperators of degree k, and the number of nodes $n_l(k)$ of degree k. Then, $P_c(k)$ is computed as $P_c(k) = \sum_l c_l(k)/\sum_l n_l(k)$.

In Fig. 3 we report $P_c(k)$ for different values of the temptation, b, and for two SF networks corresponding to the lowest and highest values of the CC. As b increases, and hence the average cooperation $\langle c \rangle$ decreases, the curves $P_c(k)$ show the same behavior in the two networks considered. In particular, high degree nodes are more resistant to defection and display the highest values of $P_c(k)$, for each value of b. In addition to this, the profile of $P_c(k)$ shows, for all the curves, a well defined global minimum for intermediate degree classes. Therefore, low connected nodes are not the easiest ones to be invaded by defectors. This result has been previously reported for BA networks in [24]. In BA networks the existence of the minimum is explained by the presence of low degree nodes (the last nodes to be attached in the network growth process) that are only connected to the hubs. These leaves are thus isolated by hubs from the rest of the network and therefore imitate and fixate the cooperative strategy adopted by their corresponding neighboring hubs. The same picture applies for highly clustered networks but with an important difference regarding the organization of leaves around hubs. In this case, the last nodes attached to the network are usually connected both with a hub and with other low degree nodes (also attached to the hub). These nodes are again dynamically isolated from the rest of the network by the hub and thus they imitate and then fixate the hub's strategy. Additionally, the links between isolated leaves that close the triads (composed of a hub and two leaves) nourish these leaves with a new mechanism to resist defection since their payoff is now provided both from the hubs and other leaves. Therefore, any eventual change of the state of the hubs is not trivially followed by a change of leaves' state since they can still obtain payoff from the interactions that share among them. In other words, the density of triangles around hubs in highly clustered SF networks enhances the fixation of cooperation in low degree nodes.

Let us now focus on the path towards $\langle c \rangle = 0$ as b increases. Although the overall picture revealed from Fig. 3 seems to be qualitatively the same regardless the CC of the networks, a careful inspection of the results reveals that a high CC tends to homogenize the role of degree classes when defectors invade the network. In Fig. 4 we have plotted again the probability $P_c(k)$ for several SF networks of different CC. In each

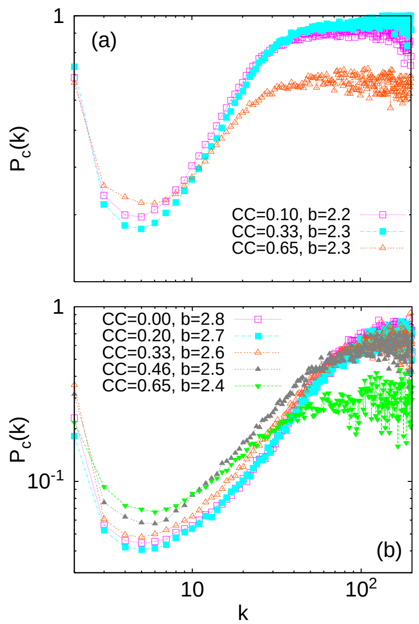

FIG. 4: Probability $P_c(k)$ of finding a node of degree k playing as co-operator in the stationary regime of the evolutionary dynamics. Each panel shows $P_c(k)$ for networks with different CC and the same average level of cooperation: **(a)** $\langle c\rangle = 0.35$, **(b)** $\langle c\rangle = 0.05$. Note that in each panel the curves $P_c(k)$ correspond to different values of the temptation to defect, b, for each network.

panel of the figure we have plotted the curves $P_c(k)$ of several networks at different temptation values, b, so that the average level of cooperation, $\langle c\rangle$, is the same in all the networks. Namely, Figs. 4.a and 4.b correspond to $\langle c\rangle \simeq 0.35$ and 0.05 respectively. For low clustering networks the shape of $P_c(k)$ can be naively described by defining a quantity $k^*(b)$, so that for $k > k^*(b)$ we have $P_c(k) \simeq 1$, while $P_c(k) \ll 1$ for $k < k^*(b)$. This description has been already introduced in [24] for BA networks. Obviously, the value $k^*(b)$ grows with b (see Fig.3.a) and hence the conversion of cooperator into defector strategies can be explained as a progressive invasion of the degree classes by defectors: the larger the value of b the more degree hierarchies defectors have invaded. This evolution points out a smooth transition towards $\langle c\rangle = 0$ for SF networks with low CC values, as reported in Fig. 2. Conversely, for highly clustered SF networks there is not such critical threshold $k^*(b)$ and the invasion by defectors affects homogeneously the degree classes. This is clear from Figs. 4.a and 4.b by looking at the curves $P_c(k)$ corresponding to SF networks with $CC = 0.65$. In these two curves, corresponding to $\langle c\rangle = 0.35$ and 0.05, all the degree classes have been already affected by the invasion of defectors. Therefore, one cannot describe the path towards $\langle c\rangle = 0$ in highly clustered SF networks as a hierarchical invasion of defectors such as in the BA case [24]. On the contrary, the degree hierarchy seems not to play a crucial role as soon as defectors invade highly clustered networks. This result would explain the sudden drop of cooperation reported in Fig. 2 for high values of CC as a consequence of the low ability of clustered networks to bias defector strategies towards low and intermediate degree classes.

Conclusions.- We have studied the role of clustering, a typical property of social systems, in the evolution of cooperation in SF networks. Our conclusion is twofold. On the one hand, a significative enhancement of cooperation is shown when the clustering coefficient of the network is high. This enhancement is manifested by the persistence of a population of (nearly) all cooperators in the network even for large values of the temptation to defect. On the other hand, the transition to zero level of cooperation becomes sharper as the clustering of the network increases. The sudden drop of the cooperation in highly clustered populations is explained as a consequence of the spreading of defector strategies across all the degree classes. Therefore, the picture of a hierarchical invasion of defectors previously observed in BA networks does not apply for highly clustered SF networks.

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